An Efficient Algorithm for the Calculation of Parasitic Coupling Between Lines in MICs

John M. Dunn and Lincoln Cole Howard

University of Colorado, Department of Electrical and Computer Engineering

Campus Box 425, Boulder, CO, 80309-0425

Abstract

A new algorithm is developed to calculate parasitic coupling between transmission lines in an efficient manner. The algorithm works by using the currents and voltages on the lines in the absence of parasitics to calculate independent voltage and current sources, which then give the approximate coupling strength between the various lines. These sources are easy to place in a CAD program. The algorithm is demonstrated on a double stub filter structure. The observed splitting of the resonance, for this particular example, is modeled by using dependent sources as calculated from the independent sources.

I) Introduction

Parasitic coupling is becoming more of a concern in microwave integrated circuits (MICs) as packing densities of components are increased and frequencies are pushed higher. By parasitic coupling, we mean unintended electromagnetic coupling between two components in the circuit. Such coupling can have a deleterious effect on the circuit's performance.

The effects of parasitic coupling are not included in commercially available circuit simulators at this time. These simulators are based on microwave network theory concepts, and they therefore only include the interaction between two circuit elements if it is specifically specified by the designer. For example, two parallel transmission lines are assumed not to be coupled unless the designer explicitly puts a coupled line model into the circuit. The designer is faced with two problems when trying to included parasitic coupling effects in the circuit design. The first issue is how to determine the strength of the coupling between two elements; the second issue is how to incorporate this effect back into the computer aided design (CAD) software in a reasonable manner.

One approach that has been tried with some success is to numerically calculate the coupling between the two elements using an electromagnetic simulator. These simulators numerically solve Maxwell's equations for the given structure. They therefore can potentially include all electromagnetic effects. This approach has two serious limitations for the circuit designer. The first is that circuit simulators are computationally very intensive, typically taking several minutes for each frequency point when computed on a reasonable sized workstation. The second problem is that the designer gets very little intuitive feeling about how important the coupling is to the circuit's performance. This makes it difficult to design around any coupling problems that may occur.

In this paper, we develop an algorithm for calculating parasitic coupling between transmission line elements which does not suffer from the drawbacks mentioned above. The algorithm is quick to implement numerically, and can give the designer a better intuitive idea of the levels of coupling to expect. The formulas are approximate and therefore subject to a number of limitations. First of all, the algorithm is only developed for microstrip transmission lines. It should be possible to extend it to other transmission line structures. Coupling between structures which are not transmission lines is outside the scope of the algorithm. The second restriction is that the lines are far enough apart that the coupling is a small effect. By this we mean that the currents and charges on the two lines without coupling included can be used as an initial guess for insertion into the algorithm. If the coupling is so strong that this is not the case, the algorithm may behave poorly. Parasitic is considered here to be a perturbation on the circuit's performance. This is usually the case in realistic examples.

The algorithm is developed in the next section. It is based on transmission line theory and the inclusion of first order coupling effects. The effect of coupling is included in the CAD program by inserting independent voltage and current sources in the transmission lines. The third section of the paper demonstrates the algorithm on a coupled stub structure. This example has been discussed a great deal in the literature. It is known to be a good example of parasitic coupling effects [1]. The algorithm gives coupling of the right strength. It does not, however, produce a splitting of the resonance in the circuit. This defect is remedied by showing how dependent sources may be used for this example, in place of independent sources. The agreement between predicted and experimental results is reasonable.

IF1

II) Derivation of the Algorithm

The algorithm is explained in this section. Assume that we have two transmission lines labeled a and b. See Figure 1. The lines are usually embedded in an MIC. We wish to calculate the parasitic coupling between line a and b. Specifically, we wish to find the effect on line b from currents and voltages existing on line a. The entire procedure can later be reversed to calculate the effect on line a from currents and charges of line b. The effect of the coupling is modeled by placing independent current and voltage sources on line b. The strength of the sources is determined by the current and charge on line a, in the absence of coupling. In practice, the CAD program is first run without coupling. The voltages and currents on the lines are determined. The independent voltage and current sources are then calculated and inserted into the circuit. The simulation is then rerun.

The voltage and current on the two lines are given by: $V_j(\zeta_j)$, and $I_j(\zeta_j)$, where: $j = a, b$. MKS units are used throughout this paper. The charge/unit length on a line is denoted by: $\rho_j\left(\zeta_j\right)$. The rectangular coordinate system is defined to have z in the vertical direction. The ground plane is at $z = 0$; the interface between the substrate and air is at: $z = h$. The relative dielectric constant of the substrate is denoted by: ϵ_r. Only single layer, non-magnetic substrates are considered in this paper, although the algorithm could , in principle, be generalized to any transmission line system for which a Green's function exists. Local coordinates along a line are denoted by: ζ_a or ζ_b. In actually carrying out the integrals derived in this paper, all local coordinates have to be converted to rectangular coordinates.

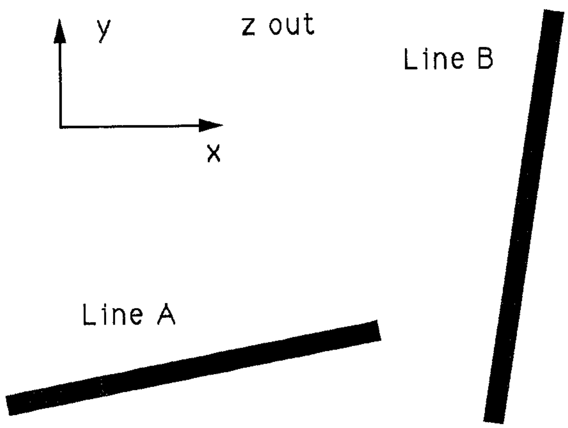

Figure 1: Geometry of the Problem

We assume that the coupling is weak in the sense that the currents and voltages on the two lines in isolation are not changed dramatically when the coupling included. It is therefore possible to use the isolated current and voltage on line a as the starting point for calculating the coupling. We also assume that the width of the lines can be neglected. All of the current and charge is lumped into an infinitesimally wide element. This restriction can be lifted , at the cost of increasing the computational time. Assume charge and current distributions on line a are known in the absence of parasitic coupling. They could be found in practice by running the circuit simulator without any parasitics included. The charge and current on the line are the source of electromagnetic fields. These fields excite charges and currents on line b. In reality, there is then an influence back on line a from line b. We ignore this complication because of our assumption of weak coupling. It is more convenient to work with the vector potential, $\vec{A}$, and scalar potential, V, than with the electromagnetic fields themselves. The voltage and vector potential due to the charges and currents of line a influence line b. The charge and current on line a produce potentials:

$$V(x,y) = \int_{linea} d\zeta_a \rho_a\left(\zeta_a\right) ge\left(\zeta_a;x,y\right)$$
$$\vec{A}(x,y) = \int_{linea} d\zeta_a \vec{x}_a I_a\left(\zeta_a\right) gm\left(\zeta_a;x,y\right) \qquad (1)$$

The unit vector, $\vec{x}_a$, is in the direction of the line a. If The line is not straight, $\vec{x}_a$, changes; this must be included in the integration. The functions ge and gm are Green's functions. They are described in detail in the published literature [2,3]. The units of ge are 1/farads. The units of gm are henrys/m^2.

Faraday's law is applied to a mathematical surface running underneath line b. After some manipulation, it can be shown that the first of the Telegrapher's equations results:

$$\frac{\partial V_b}{\partial \zeta_b} + j\omega \ell_b I_b\left(\zeta_b\right) = -j\omega A\left(\zeta_b\right) \qquad (2)$$

where the source term A is given by the amount of vector potential, $\vec{A}$, from the current of line a, in the direction of line b: $A = \vec{A}\cdot\vec{x}_b$. The inductance/unit length of line b is given by ℓ_b. The second Telegrapher equation can be derived by forcing conservation of charge on line b. It can be shown that this reduces to:

$$\frac{\partial I_b}{\partial \zeta_b} + j\omega c_b V_b\left(\zeta_b\right) = j\omega c_b V\left(\zeta_b\right) \qquad (3)$$

The capacitance/unit length of line b is c_b.

Equations (2) and (3) are solved for V_b and I_b, using standard techniques. The line is assumed to be terminated in its characteristic impedance. It is found that:

$$V_b(\zeta) = \frac{\gamma_b}{2}\int_{lineb} d\zeta_b \left[V\left(\zeta_b\right) + \frac{j\omega}{\gamma_b^2}\frac{\partial A}{\partial \zeta_b}\right] e^{-\gamma_b|\zeta-\zeta_b|}$$
$$I_b(\zeta) = \frac{\gamma_b}{2\ell_b}\int_{lineb} d\zeta_b \left[-A\left(\zeta_b\right) + \frac{j}{\omega}\frac{\partial V}{\partial \zeta_b}\right] e^{-\gamma_b|\zeta-\zeta_b|} \qquad (4)$$

In general, there is also a nonzero, homogeneous solution to equation (4). It is always possible that voltages and currents existed on line b from sources other than line a. The procedure followed here for line a would then have to be repeated for line b. Such charge and currents would be a source of charge and current on line a.

Equation (4) is not in a convenient form for insertion into a CAD program. Instead, we now calculate equivalent voltage and current sources on line b that give the same response at the ends of the line, as that predicted by equation (4). Once found, the current and voltage sources can be inserted into a CAD program a straightforward manner. A shunt current source is inserted into the line at: $\zeta = s_I$. A series voltage source is inserted at $\zeta = s_V$. See Figure 2. It can be shown

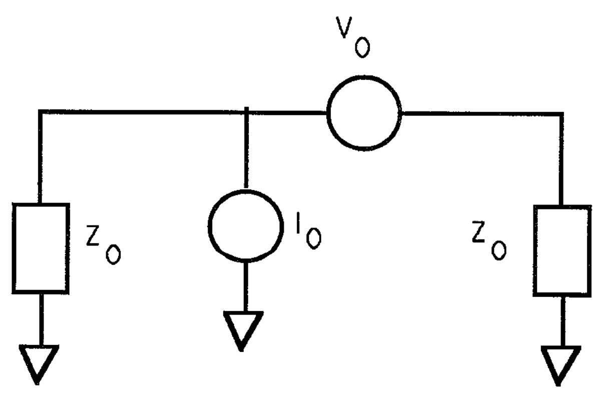

Figure 2: Independent Current and Voltage Sources

that the strength of the current and voltage sources are given by:

$$V_0 = \delta \int_{lineb} d\zeta_b \left(\gamma_b V\left(\zeta_b\right) \sinh\left[\gamma_b\left(\zeta_b - s_I\right)\right] -j\omega A\left(\zeta_b\right)\cosh\left[\gamma_b\left(\zeta_b - s_I\right)\right]\right)$$

$$Z_0 I_0 = \delta \int_{lineb} d\zeta_b \left(\gamma_b V\left(\zeta_b\right) \cosh\left[\gamma_b\left(\zeta_b - s_V\right)\right] -j\omega A\left(\zeta_b\right)\sinh\left[\gamma_b\left(\zeta_b - s_V\right)\right]\right) \quad (5)$$

where:

$$\delta = \frac{1}{\cosh\left[\gamma_b\left(s_V - s_I\right)\right]} \quad (6)$$

Equation (5) is the final, desired equation.

III) Example of Two Coupled Stubs

In this section, the algorithm is applied to a simple stub filter structure. The circuit is shown in Figure 3. The structure has been studied extensively [1], and is known to be sensitive to parasitic coupling. The circuit was built on Alumina substrate, with: $\epsilon_r = 9.6$; substrate thickness, $h = 125\mu m$; and line widths of 122 μm. This makes the characteristic impedances of all lines close to 50Ω. The stub lengths are 2.921 mm. The separation between the stubs was varied, with experimental values ranging from 0 to 757 μm. Separation is defined to be the center-to-center distance.

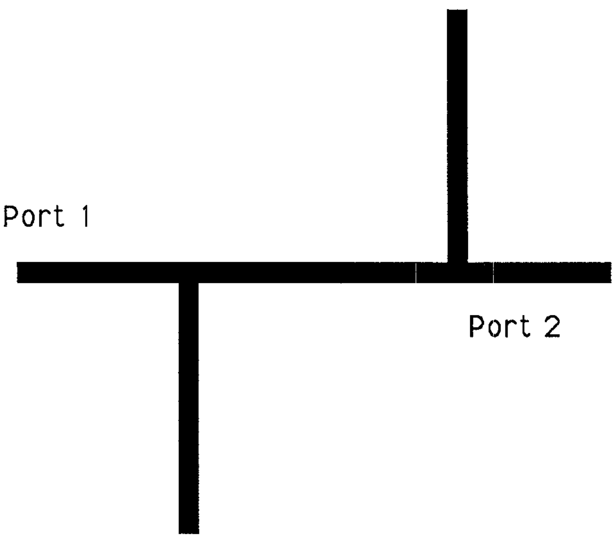

Figure 3: Double Stub Microstrip Filter Structure

The theoretical and measured results are compared for a separation between the stubs is 757 μm. The theoretical results were calculated using a commercially available program [4]. The substrate is assumed to be lossless; the lines are made of gold, and are 1.5 μm thick. There are a number of discrepancies between the measured and simulated results. The most dramatic discrepancy is a "double-dip" in $|S_{21}|$ near 10 GHz, which appears in the experimental results, but not the simulated. This region is shown in more detail in Figure 4. This splitting of a resonance is characteristic of coupled resonators. Parasitic coupling between the stubs is therefore suspected.

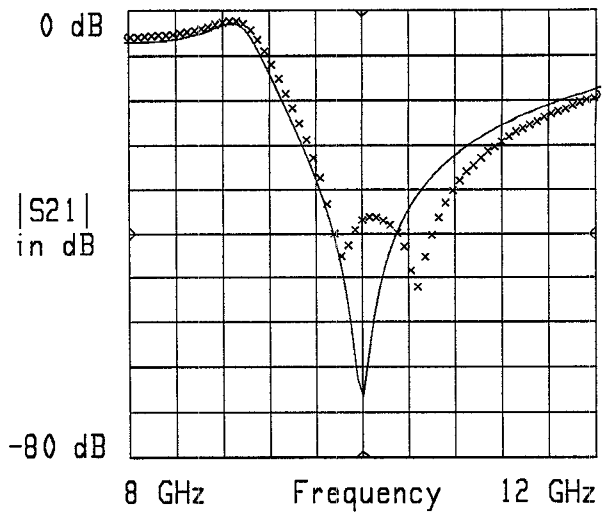

Figure 4: Theoretical and Measured $|S_{21}|$

It is first of all necessary to calculate the voltage and current without coupling. We do so at the resonance of the stubs, where the parasitic coupling is expected to be important. At resonance, the stubs are 1/4 of a wavelength long; they appear as shorts to the main line. It is therefore assumed that the incident wave from port 1 is entirely reflected back. Furthermore, a current and voltage exist on the left stub. They are given by:

$$I(\zeta) = I_m \frac{\sin\left[\beta_e\left(\zeta + \zeta_s\right)\right]}{\sin\left[\beta_e \zeta_s\right]}$$

$$V(\zeta) = V_m \frac{\sin\left[\beta_e \zeta\right]}{\sin\left[\beta_e \zeta_s\right]} \quad (7)$$

The wavenumber is β_e. The stub length is ζ_s. The maximum current, I_m, occurs at the beginning of the stub. The maximum voltage, V_m, occurs at the end of the stub. The current and voltage are not independent, but are related by the Telegrapher's equations. These give:

$$V_m = -jZ_c I_m \quad (8)$$

where Z_c is the characteristic impedance of the stub. The structure is excited by putting 1 watt of power into port 1. The S-parameters can be easily calculated from the transmitted and complex power.

It was decided to put the independent voltage source at the beginning of the second stub, and the independent current source at the end of the second stub. This was done in order to be able to later change to dependent sources, as will be described in detail below. It is not possible to use equation (5) to calculate I_0 and V_0, because the cosine terms in the denominators go to zero at resonance. It was necessary to calculate the strength of the sources by first of all recalculating equation (4) for the case where line b is terminated in a short at one end and an open at the other. The equivalent of equation (5) was then found by finding the independent sources that give the right voltage and current at the ends of the line. The final result is similar to equation (5):

$$V_0 = \int_0^{\ell} d\zeta_b \left(\gamma_b V\left(\zeta_b\right) \sinh\left[\gamma_b \zeta_b\right] -j\omega A\left(\zeta_b\right)\cosh\left[\gamma_b \zeta_b\right]\right)$$

$$Z_0 I_0 = \int_0^{\ell} d\zeta_b \left(\gamma_b V\left(\zeta_b\right) \cosh\left[\gamma_b \zeta_b\right] - j\omega A\left(\zeta_b\right) \sinh\left[\gamma_b \zeta_b\right]\right) \quad (9)$$

The results of the simulation are shown in Figure 5.

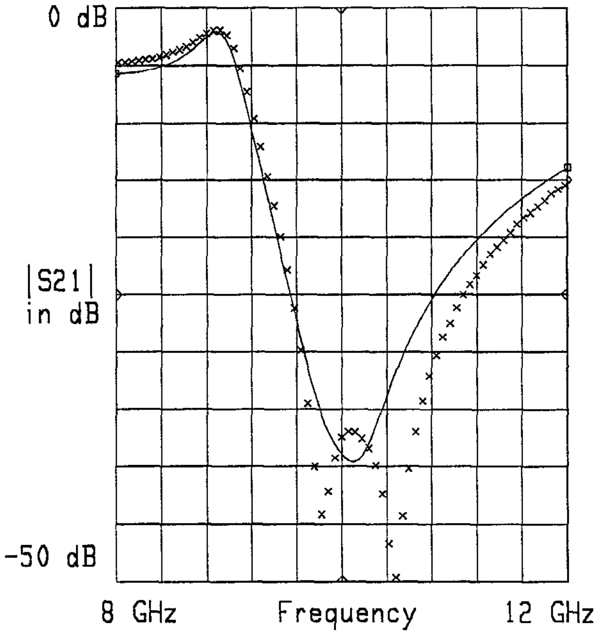

Figure 5: Independent Source Simulation Results

Equation (9) was used to calculate V_0 and I_0. We found: $I_0 = 0.000190 + j0.00115$ A, and $V_0 = 0.00952 + j0.0572$ V. The parasitic coupling has brought up the level of the dip near 10 GHz to experimental values. It has not, however, given a splitting of the resonance. This is because we are inserting independent sources into a linear system. The splitting of a resonance can only be caused by changing the linear system itself, not its excitation. Fortunately, the system can be modified to account for this problem. The idea is to insert two dependent sources in the second stub, instead of using an independent voltage and current source. In fact, we will use a mutual inductance and capacitance. Capacitance can be regarded as a dependent current source; inductance as a dependent voltage source. This is why the sources were placed at the ends of the stub. It is the only place where the current source can be clearly attributed to a mutual capacitance, and the voltage source to a mutual inductance. The strength of the capacitance and inductance are found by knowing the current and voltage strength needed, and the current and voltage at the beginning and end of the first stub.

Figure 6 shows the results of the simulation when the calculated C_m and M are used for a stub separation of 757 μm. Values of $C_m = 1.30 \times 10^{-15} - j2.14 \times 10^{-16}$ F and $M = 3.22 \times 10^{-12} - j5.36 \times 10^{-13}$ H were used. (The inductance was given an extra 180 degree phase shift from the value listed here when inserted into the CAD program. This is because the mutual inductance in the circuit program has a voltage sign convention opposite to that given by Faraday's law.) If the simulated results are shifted by 150 MHz, the curves lie on top of one another. This slight frequency shift is considered to be quite reasonable considering the approximate method used. The agreement is also good for smaller separations.

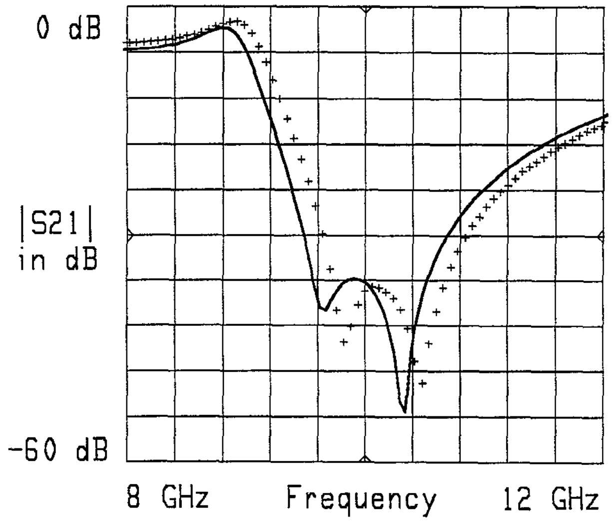

Figure 6: Dependent Source Results for 757 μm Separation

Errors similar to the one seen in Figure 6 were seen for separations down to 249 μm, corresponding to a separation of two substrate thicknesses.

References

[1] M. Goldfarb, and A. Platzker,"The effects of electromagnetic coupling on MMIC design", **Microwave and Millimeter Wave CAE**, vol. 1, no. 1, pp. 38–47, Jan. 1991.

[2] J. Mosig, " Integral equation technique", chapter 4, **Numerical Techniques for Microwave and Millimeter-Wave Passive Structures**, ed. by T. Itoh, New York: J. Wiley and Sons, 1989.

[3] J. M. Dunn, "A uniform asymptotic expansion for the Green's functions used in microstrip calculations", **IEEE Trans. Microwave Theory Tech.**, vol. MTT–39, pp. 1223–1126, July, 1991.

[4] Hewlett-Packard Microwave Design System, Hewlett-Packard Co., Santa Rosa, CA

[5] K. C. Gupta, R. Garg, and R. Chadha, **Computer-Aided Design of Microwave Circuits**, Artech House, Dedham, MA, 1981.